Business Cycles, Worker Flows, and Hiring Discrimination

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Preliminary and Incomplete. This version: Oct, 2020 The most recent version is available at *http://luischanci.com*

Abstract

What does hiring discrimination against black Americans imply for unemployment, job finding, and separation rates over the business cycle? Using a searchand-matching model with endogenous separations and an urn-ball matching function, this paper shows that discrimination in the early stages of the hiring process leads to adverse labor outcomes over the business cycle: lower job finding probabilities and higher unemployment volatility for discriminated groups. Conversely, the model does not predict significant differences in separation rates.

JEL codes: E24, E32, J64, J71. Keywords: Unemployment, Discrimination, Business Cycle.

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1 Introduction

Resume studies have researched labor-market discrimination against racial, religious, and ethnic groups. In those studies, several applications of similar fictitious candidates are sent out to real job openings. Stated briefly, candidates have similar observable characteristics that explain productivity but differ in only one demographic characteristic (e.g., race, sex, or religion). Given their quasi-experimental research design, results in those studies are considered strong evidence for labor-market discrimination (see, for instance, Bertrand and Mullainathan, 2004; Rich, 2014; Bertrand and Duflo, 2017).

Starting from the evidence that there is racial discrimination during the hiring process, in this paper I contribute to the empirical literature on Labor Economics and Macroeconomics by studying the following research question: What does hiring discrimination against black Americans imply for unemployment, job-finding, and separation rates over the business cycle? To address the research question, I first motivate the topic with empirical evidence using data from the Current Population Survey (CPS). Here, I assess the relative volatility of different racial groups' labor outcomes over the business cycle. I then present a search-and-matching model of the labor market to study the effects of hiring discrimination. The model has two relevant modifications with respect to a baseline model (e.g., Pissarides, 2000): (1) as in Mortensen and Pissarides (1994) and Fujita and Ramey (2012), the model has endogenous job destruction; and (2) the matching function allows for different degrees of hiring discrimination, following the modified urn-ball matching function in Kuhn and Chancí (2019).

Kuhn and Chancí (2019) explore the business cycle consequences of hiring discrimination for different demographic groups (i.e., based on race or sex). The authors provide a mapping from the degree of hiring discrimination to the cyclicality of labor market outcomes by embedding a modified urn-ball matching function into a searchand-matching model. The idea behind the primary mechanism in the model is that during recessions, when unemployment is high, the pool of applicants is larger and, therefore, the relative probability that a majority worker will be picked over a minority worker is higher. Their results suggest that discrimination in the hiring process leads to higher unemployment volatility during the business cycle.

Furthermore, the authors use a standard search-and-matching model where separations are constant across worker types and over time. This framework means that fluctuations in the job-finding rate mainly drive the volatility in unemployment. Some empirical evidence suggests, however, that fluctuations in separation rates play an important role in explaining the variations in unemployment. For instance, Fujita and Ramey (2009), using CPS data, show that around 50 percent of the fluctuations in unemployment can be attributed to variations in the separation rates. This evidence raises the question of whether modeling separation as an additional source of fluctuations in Kuhn and Chancí (2019) may affect the conclusions concerning the volatility of the unemployment of the discriminated groups.

In this paper, I extend the work in Kuhn and Chancí (2019) to introduce endogenous separations into the search-and-matching model. This variation makes the model as a whole more realistic and adds an additional margin along which heterogeneity can be observed, meaning the possibility to study workers flows and separation rates. The modification, however, involves two challenges: (1) the definition of equilibrium, and (2) setting a suitable numerical solution method. I show that the equilibrium conditions are based on the relationship between (un)employment and vacancies. Moreover, I describe a numerical solution that relies on the value function iteration method. This technique represents an alternative to the perturbation method used in Kuhn and Chancí (2019). Thus, the model itself is a contribution to the literature. The combination of labor market frictions, heterogeneous agents, and endogenous separations may also be calibrated to study other questions in labor economics where agents differ in their job-finding probabilities (e.g., due to differences in the level of education).

As in Pissarides (2000) and Fujita and Ramey (2012), I endogenize the job destruction decision by considering both aggregate and idiosyncratic productivity shocks. The combination of these shocks produces some levels of productivity at which production is not profitable, and, therefore, firms destroy jobs. Thus, in comparison to a model with a constant separation rate, including endogenous separations produces more realistic volatility and productivity responsiveness of the separation rate and worker flows (Fujita and Ramey, 2012).

To allow for hiring discrimination, I implement the matching function in Kuhn and Chancí (2019). This function is set up for two types of workers, and the difference in conditional hiring discrimination between them is basically summarized in one parameter. This parameter is the relative likelihood that one type of worker gets a job. Hence, point estimates for differentials in callback rates found in resume studies can be used to calibrate this parameter in the matching function of the model.

I calibrate the model to match the aggregate labor market statistics of the U.S. economy. Only a subset of the model parameters are directly assigned, and the rest are estimated by Simulated Method of Moments. Comparing the results from the model

with estimates from a linear probability model using CPS data, the model can explain about 80 percent of the extra business cycle volatility in the unemployment rate of black workers. I then use the model for a counterfactual experiment. I study the effects of a reduction in discrimination by half, meaning an improvement in the relative hiring probability of black workers. The results show significant improvements in the labor market outcomes of this discriminated group. The excess of unemployment volatility of black workers disappears, and the gap in job-finding rates decreases by about 50 percent. Finally, I do find that the separation rates are not constant throughout the business cycle, but do not find significant variations across groups after the reduction in discrimination.

Following this introduction, Section 2 provides empirical evidence on unemployment dynamics by race using CPS data. Section 3 outlines the model, which is then calibrated in Section 4. This section also presents the main simulation results, a discussion of the results, and conducts a counterfactual exercise for hiring discrimination. Finally, Section 5 concludes.

2 Empirical Evidence

It is a well-known empirical fact that labor outcomes differ by race. This subsection reviews some of the empirical evidence that motivates the research question in this paper. The empirical results will also be an essential tool to evaluate the theoretical model later in the quantitative section.

2.1 Data

I use the Current Population Survey (CPS) to assess the relative volatility of different groups' labor outcomes over the business cycle. The CPS is the primary source of labor force statistics for the population of the United States. It is a monthly sample of individual workers, determining whether they are employed and, if non-employed, whether they engaged in an active job search activity. The survey is a rolling panel of housing units, which are surveyed according to a 4-8-4 pattern: Residents remain in the data set for four consecutive months, drop out for the following eight months,

and then are interviewed again for four months. Within a household, all persons are surveyed.

I use monthly data for the period 1984m01-2018m01 from the IPUMS-CPS, University of Minnesota (Flood et al., 2018). I follow Cajner et al. (2017), Kuhn and Chancí (2019), and Hoynes et al. (2012), to construct the final database to use in my research. To illustrate, I exclude observations for people younger than 25 or older than 55 years old, who are retired or who are members of the armed forces. Furthermore, I focus on blacks or African Americans because it is one of the central groups researched in resume studies, and because, as in Kuhn and Chancí (2019), race is a characteristic that is straightforward to work with empirically¹.

The final data set contains more than twenty-four million observations, with black Americans representing about 10 percent of the total sample. Table 1 presents summary statistics of the unemployment rate by sex, age, and levels of education. The figures in this table show that African Americans have higher unemployment rates. For instance, the unemployment rate of young blacks (25-29) and black Americans with low levels of education (Some high school) are about 13 and 17 percent, respectively. These numbers overcome more than two times the national long-term unemployment rate of other demographic groups. Furthermore, the gap in unemployment rates between black and white workers is more than four percent. Finally, it does not seem to be that there are substantial differences in the unemployment rates by sex.

2.2 Empirical strategy

The objective is to compute estimates of the additional volatility in labor outcomes of black Americans, with respect to other groups, during different phases of the business cycle. Thus, the empirical strategy is a linear probability model that controls for observable characteristics. Although observational data may not allow one to perfectly mimic the setup of resume studies, which assign race randomly and hold all other factors constant, given the available information, this approach offers the first best option. Thus, the model includes several observable characteristics that are captured in the CPS.

There are three labor outcomes that are relevant to the model: unemployment rates, job-finding rates, and separation rates. The linear probability model approaches these

¹It is directly measured in the CPS (unlike, for instance, sexual orientation).

	White	Black	Hispanic	Other	Overall
	Mean/(S.d.)	Mean/(S.d.)	Mean/(S.d.)	Mean/(S.d.)	Mean/(S.d.)
Panel A. Sex					
Male	4.13	9.40	6.15	5.04	5.00
	(19.9)	(29.2)	(24.0)	(21.9)	(21.8)
Female	3.93	8.62	7.24	4.96	4.97.
	(19.4)	(28.1)	(25.9)	(21.7)	(21.7)
Panel B. Age					
25-29	5.25	12.79	7.65	6.43	6.66
	(22.3)	(33.4)	(26.6)	(24.5)	(24.9)
30-34	4.39	10.35	6.73	5.16	5.52
	(20.5)	(30.5)	(25.1)	(22.1)	(22.8)
35-39	3.91	8.64	6.43	4.74	4.87
	(19.4)	(28.1)	(24.5)	(21.3)	(21.5)
40-44	3.64	7.51	6.11	4.26	4.41
	(18.7)	(26.4)	(23.9)	(20.2)	(20.5)
45-49	3.54	6.88	5.97	4.60	4.21
	(18.5)	(25.3)	(23.7)	(21.0)	(20.1)
50-55	3.57	6.46	6.08	4.75	4.14
	(18.5)	(24.6)	(23.9)	(21.3)	(19.9)
Panel C. Schooling					
Some high school	9.34	16.61	8.72	9.56	10.08
0	(29.1)	(37.2)	(28.2)	(29.4)	(30.1)
High school or GED	5.00	10.35	6.53	6.51	6.02
0	(21.8)	(30.5)	(24.7)	(24.7)	(23.8)
Some college	4.02	7.82	5.47	5.85	4.79
Ũ	(19.6)	(26.8)	(22.7)	(23.5)	(21.4)
Bachelor's degree	2.64	4.57	3.93	3.72	2.99
0	(16.0)	(20.9)	(19.4)	(18.9)	(17.0)
Higher degree	1.97	3.44	2.75	2.69	2.20
0 0	(13.9)	(18.2)	(16.4)	(16.2)	(14.7)
Panel D. Overall	4.04	8.99	6.59	5.00	4.99
	(19.7)	(28.6)	(25.0)	(21.8)	(21.8)
Observations	17,925,300	2,429,621	2,364,001	1,433,109	24,152,031

Table 1: Descriptive Statistics. Unemployment Rate (%) by Groups.

Notes: This table reports the mean value and the standard deviation (in parentheses) for the unemployment rate, in percentage, by groups over 1984m1-2018m3. Calculations using the Current Population Survey (CPS). Observations are weighted using CPS final weights. variables using the following three state dummy variables as dependent variables. Unemployment takes the value of one if the individual does not have, but is looking for, a job. Job-finding is equal to one if the worker found a job while unemployed and separation is equal to one if the individual switches from employed to unemployed.

On the other hand, as a measure of the state of the business cycle, the linear model uses the aggregate unemployment rate as provided by the Bureau of Labor Statistics. Thus, all the independent variables are included in levels and also as the interaction with the aggregate unemployment variable.

Equation (1) summarizes the main regression specification for an individual i at time t

$$y_{it} = \gamma_1 \text{Black}_{it} + \gamma_2 (\text{Black}_{it} \times u_t) + \mathbf{x}'_{it} \mathbf{\beta}_1 + (u_t \times \mathbf{x}'_{it}) \mathbf{\beta}_2 + \varepsilon_{it}$$
(1)

where *y* is labor outcome of interest; *u* is the aggregate unemployment rate; *Black* is a dummy variable for membership in the respective demographic group as defined by the CPS; *x* is a vector of controls (including a constant), and ε is a random noise component. The controls include (a quadratic in) age, educational achievement², family status³, city size and occupation in the detailed categories provided by the CPS. The latter variable refers to the occupation in which the individual is currently employed or, in case of unemployment, held last. Additional robustness checks include year effects, industry effects, and other variables.

In equation (1), the coefficient of interest is the interaction of the race dummy with the unemployment rate, γ_2 . This coefficient indicates how much the group-specific likelihood of employment, conditional on the demographic variables, increases relative to a white male's likelihood of unemployment when the aggregate unemployment rate increases by one percentage point. Again, given that the interest is γ_2 , all controls are included both in levels and as interaction with the aggregate unemployment rate. Observations are also weighted using CPS final weights.

Table 2 presents the results for equation (1). The columns describe the dependent variable used in each specification, and the rows the estimates of γ_1 and γ_2 .⁴ Results in columns 1 and 3 are similar to the results in Kuhn and Chancí (2019) and results in column 2 provide additional empirical evidence on the separations rate.

²Less than high school, high school, some college, college, post-graduate degree.

³Married without children, married with children, unmarried without children, unmarried with children.

⁴Additional estimates are omitted.

	Unemployment Outflow Rate Rate (Separation Rate)		Inflow Rate (Job-finding Rate)	
Black	0.0016	0.0035**	-0.0568***	
	(0.0009)	(0.0005)	(0.0066)	
Black \times Unemployment	0.0039***	0.0004***	0.0027**	
	(0.0002)	(0.0001)	(0.0010)	
<i>R</i> ²	0.042	0.010	0.060	
Observations	17,939,045	15,492,110	612,034	
Controls	Yes	Yes	Yes	

Table 2: Unemployment, Worker Flows, and Business Cycle.

Notes: The table reports the results for equation (1). The first row presents the names of the dependent variable in each specification. Additional estimates are omitted. Using the Current Population Survey (CPS) over 1984m1-2018m3. Observations are weighted using CPS final weights. Robust standard errors in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

Overall, the results have the expected sign and significance. Only the specification for Job-finding suggests that the gap between blacks and whites does *not* significantly widen in a recession. There is a significant difference in the baseline levels of the job-finding rate, as evidenced by the sizeable negative coefficient on the Black dummy. As in Kuhn and Chancí (2019), this is consistent with the theoretical model: Due to the lower baseline level of job-finding rates for blacks, smaller fluctuations can have a relatively more substantial impact on their absolute unemployment numbers.

The coefficient of interest, Black \times Unemployment, has values of 0.0039, 0.0004, and 0.0027, in columns 1, 2, and 3, respectively. These numbers mean that, on average, for every percentage point increase in aggregate unemployment, the unemployment rate, the separation rate, and the job-finding probability rate of black Americans increases about 0.4, 0.04, and 0.27 percentage points, respectively. In other words, to put the result of unemployment into more context, let's say that, for instance, aggregated unemployment increases from 5 to 10 percent (similar to the most recent financial crisis). Using average values for the other control variables, coefficients in column 1 mean that the gap of unemployment for blacks with respect to whites will sharply increase from 2 to 4 percent. The result represents an economically substantial effect and is also consistent with the fact that resume studies find significant differences in

callback rates for black people.

3 Model

This subsection presents a stochastic equilibrium model of the labor market. The model extends the canonical search-and-matching model (e.g. Pissarides, 2000) in two directions. First, as in Kuhn and Chancí (2019), the matching function corresponds to an urn-ball function with two types of agents. The set up facilitates the inclusion of differences in conditional hiring discrimination between groups, which is later calibrated with empirical evidence from resume studies. Second, I follow Mortensen and Pissarides (1994) and Fujita and Ramey (2012) to extend the model in Kuhn and Chancí (2019) to include endogenous separations. Thus, I endogenize the job destruction decision by adding both aggregate and idiosyncratic productivity shocks. The combination of these shocks produces different levels of productivity, creating the lower level production that is not profitable, and, therefore, firms destroy jobs.

In brief, the search-and-matching model in this paper involves two types of agents and also endogenous separations. This novel combination of heterogeneous agents and endogenous separations makes the model more realistic, but, at the same time, it involves a challenge to close the model given the large dispersion in employment that is generated. I begin this section describing the environment in which agents interact, then turn to the equilibrium of the economy, the agents' decision rules, and the transition flows.

3.1 Environment

Time is discrete and continues forever. The economy is populated by a unit mass of atomistic workers and an infinite mass of atomistic firms. Workers and employers are infinitely-lived, risk-neutral and discount future values at the same rate $\beta \in (0, 1)$. In any time period *t*, a worker of type *i* may be either employed or unemployed, while a firm may be matched with a worker *i*, unmatched and posting a vacancy, or inactive. Employed workers earn wage *w*, whereas unemployed workers receive a flow benefit of *b* per time period. In general, *b* includes the total value of leisure, home

production, potential unemployment benefits, saved work-related expenditures, and it is net of job-searching cost. Firms that post vacancies pay a posting cost of *c* per time period. Workers are heterogeneous in terms of their productivity on the job. There are $h \ge 2$ match-specific productivity factors indexed by $x \in X = \{x_1, x_2, ..., x_M\}$, where $x_1 < x_2 < ... < x_M = x_h$.

Matching technology Kuhn and Chancí (2019) generalize Blanchard and Diamond (1994)'s setup to allow for arbitrary degrees of discrimination. In Blanchard and Diamond (1994), whenever two workers of different groups compete for the same vacancy the worker from the preferred group *always* gets the job. In contrast, in Kuhn and Chancí (2019) if the two workers are in the same applicant pool, the worker from the preferred group has a higher chance of getting the job (but not necessarily an infinitely higher chance). This approach captures the degree of discrimination as the relative hiring probability between two candidates conditional on being in the same applicant pool, which can be assigned to a key parameter in the model. Thus, this parameter has the same interpretation as the object of interest in resume studies, except that in those studies it is not the relative hiring probabilities that are directly observable but the relative probabilities for callbacks.

The meeting function is given by an urn-ball matching technology, where every application by a worker is represented by a ball and every vacancy by an urn. Every period, in the application stage every unemployed worker submits one application to one of the posted job openings at random. If there are many urns and balls, a law of large numbers guarantees that there is a fixed distribution of balls across urns; in other words there will be a certain fraction of urns with zero balls, a certain fraction of urns with exactly one ball, and so on. Once all applications have been assigned to employers in this way, all employers who have received at least one application hire one of the applicants by drawing one ball out of the respective urn. An employer will pick between applicants of the same group with equal probability. On average, however, employers have a bias to hire from one of the two groups. Let π be the relative probability that a given worker from one group is picked for a job relative to a given worker from the other group. For example, let $\pi = 2$. Consider an applicant pool that contains a white worker, Jack, in group 1; and a black worker, John, in group 2. Thus, Jack's chances of getting the job are twice as high as John's, independent of the size and makeup of the remaining applicant pool.⁵

 $^{^{5}}$ In the simplest case where Jack and John are the only candidates this implies that respective hiring probabilities are 2/3 and 1/3.

Thus, as Kuhn and Chancí (2019) show, the job-finding probabilities for each type $i \in \{1, 2\}$ of worker is given by

$$p_i(\theta_1, \theta_2) = \theta_i \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left(\frac{e^{-(1/\theta_1 + 1/\theta_2)}}{\theta_1^{k_1} \theta_2^{k_2} k_1! k_2!} \right) \left(\frac{\mathbb{1}_i * k_1}{\pi k_1 + k_2} \right)$$
(2)

where $\mathbb{1}_i = \pi$ if i = 1, and $\mathbb{1}_i = 1$ if i = 2; k_i stands for a large number of workers of type i; and θ_i denotes the ratio between vacancies v to unemployment u_i for type i, also defined as labor-market tightness ratio.

Hence, given the parameter π , the respective job-finding probabilities can be calculated knowing the number of vacancies and the number of unemployed workers of both groups (and hence both market tightnesses θ_1 and θ_2).

Production technology Production requires the creation of a match between a worker and an employer. Upon the creation of a match, a worker-firm match can produce an output level z * x during time period t, where z is an aggregated productivity factor that is determined according to the following exogenous process:

$$\ln z' = \rho_z \ln z + \epsilon'_z \tag{3}$$

where ϵ_z is an independent and identically distributed normal disturbance with zero mean and standard deviation σ_z . I use primes to denote next period variables.

3.2 Agents' Problem

This subsection describes the Bellman equations for the individual employer and worker that need to be satisfied in equilibrium. Let $J_i(z, x)$ indicate the values of a filled job for a given x and worker type i, and let $W_i(z, x)$, $U_i(z, x)$, and V(z, x), be the values received by an employed worker, an unemployed person and a vacancy-posting firm, respectively.

Searching stage As derived in the previous section, their job-finding rate is given by p_i and depends on both market tightnesses. Also, let $\mathbb{E}_z \{\cdot\}$ be the expectations operator with respect to the aggregate state in time period *t*. Thus, at the searching stage, the unemployed value is defined by the following Bellman equation

$$U_{i}(z,x) = b + \beta \mathbb{E}_{z} \left\{ p_{i}(\theta_{1}(z),\theta_{2}(z)) W_{i}(z',x_{h}) + (1 - p_{i}(\theta_{1}(z),\theta_{2}(z))) U_{i}(z',x) \right\}$$
(4)

An unfilled posted vacancy costs a firm an amount *c* per period and gets filled with a worker *i* with probability $q_i(\theta)$. Since all employers with at least one job applicant will match with a worker, the function

$$q(\theta_1(z), \theta_2(z)) = 1 - e^{-(1/\theta_1(z) + 1/\theta_2(z))}$$

is simply one minus the probability that no workers get matched to the employer. Hence, the value of a vacancy is

$$V(z,x) = -c + \beta \mathbb{E}_z \left\{ \sum_{i=1}^2 q_i(\theta_1(z), \theta_2(z)) J_i(z', x_h) + (1 - q(\theta_1(z), \theta_2(z))) V(z', x') \right\}$$
(5)

Production stage and endogenous separations All new matches start at $x = x_h$, but the value of x may switch in subsequent time periods. At the end of a time period, a switch occurs with a probability λ . In the latter event, the value of x for the next period is drawn randomly, according to the c.d.f. G(x). With probability $(1 - \lambda)$, x maintains its time period t value into the next time period, thus, at the production stage, the value of a match with a worker of type i is described by the following Bellman equation

$$J_i(z, x) = \max\{V(z, x), J_i^c(z, x)\}$$
(6)

where $J_i^c(z, x)$ represents the value for a firm after continuation of the match is chosen:

$$J_{i}^{c}(z,x) = zx - w_{i}(z,x) + \beta \mathbb{E}_{z} \left\{ (1-s) \left(\lambda \int_{0}^{x_{h}} J_{i}(z',y) dG(y) + (1-\lambda) J_{i}(z',x) \right) \right\} + \beta \mathbb{E}_{z} \left\{ s V(z',x') \right\}$$
(7)

and a worker of type *i* has the value function

$$W_i(z, x) = \max \{ U_i(z, x), W_i^c(z, x) \}$$
(8)

where $W_i^c(z, x)$ represents the value for a worker after continuation of the match is chosen:

$$W_{i}^{c}(z,x) = w_{i}(z,x) + \beta \mathbb{E}_{z} \left\{ (1-s) \left(\lambda \int_{0}^{x_{h}} W_{i}(z',y) dG(y) + (1-\lambda) W_{i}(z',x) \right) \right\} + \beta \mathbb{E}_{z} \left\{ s U_{i}(z',x') \right\}$$
(9)

note that *s* captures an exogenous component of the separation rate.

Wage setting The wage is determined via bilateral Nash-bargaining according to

$$w(z,x) = \arg \max \left(W_i(z,x) - U_i(z,x) \right)^{1-\eta} \left(J_i(z,x) - V(z,x) \right)^{\eta}$$
(10)

where $0 \le \eta \le 1$ is the employer's bargaining power.

The solution to this Nash-bargaining problem is such that the worker and the firm receive payoffs of $W_i = (1 - \eta)S_i(z, x) + U_i(z, x)$ and $J_i = \eta S_i(z, x) + V(z, x)$, respectively, where the match-surplus is given by $S_i = (W_i - U_i) + (J_i - V)$.

3.3 Equilibrium

As it is standard in the literature, I consider a free-entry equilibrium where the value of a job vacancy is zero at all times, V(z, x) = 0. Thus, the recursive equilibrium

can better be characterized by a Bellman equation for the surplus and a free-entry condition for vacancy posting in terms of surplus.

First, using the results from the Nash-bargaining wage setting, equation (5) can be expressed in terms of surplus

$$c = \beta \eta \mathbb{E}_z \left\{ \sum_{i=1}^2 q_i(\theta_1(z), \theta_2(z)) * S_i(z', x_h) \right\}$$
(11)

this equation is the free-entry condition and states that employers post job vacancies up to the point where the expected cost equals the expected benefit of opening and maintaining a job vacancy. Furthermore, by writing the definition of both market tightnesses in terms of the unemployment rate u_i as

$$\theta_1(z) \, u_1(z, x) = \theta_2(z) \, u_2(z, x) \tag{12}$$

it is possible to obtain the value of θ_i for every period for a given realization of the *z* process.

Second, using equations (4), (6), and (8), the Bellman equation for match surplus is

$$S_i(z, x) = \max\{0, S_i^c(z, x)\}$$
(13)

where $S_i^c(z, x)$ represents the value of match surplus after continuation of the match is chosen:

$$S_{i}^{c}(z,x) = zx - b + \beta \mathbb{E}_{z} \left\{ (1-s) \left(\lambda \int_{0}^{x_{h}} S_{i}(z',y) dG(y) + (1-\lambda)S_{i}(z',x) \right) \right\} - \beta (1-\eta) p_{i}(\theta_{1}(z),\theta_{2}(z)) \mathbb{E}_{z} \left\{ S(z',x_{h}) \right\}$$
(14)

which determine the equilibrium paths of S(z, x) for given realizations of the *z* process.

In brief, in this economy an equilibrium is a schedule of market tightness, and unemployment, for each type of worker, such that the free entry condition 11, the match surplus 13, and the vacancy condition 12 are satisfied.

3.4 Worker transition Flows and Rates

A worker who is unemployed in time period t becomes employed next period with probability $p_i(\theta_1(z), \theta_2(x))$, thus the measured number of Unemployed-to-Employed (*UE*) next period is

$$UE'_i = p_i(\theta_1(z), \theta_2(x)) u_i(z, x)$$
(15)

Separation rates and Employment-to-Unemployment (EU_i) flows depend on the distribution of x across existing matches. Let $e_i(z, x)$ be employment for a given x, there exists a value R(z), such that $S_i(z, x) = 0$ if and only if $x \leq R$. Thus, the employment distribution evolves according to

• If
$$x \leq R(z)$$
,
 $e_i(z', x) = 0$ (16)

• If
$$x \in (R(z), x_h)$$
,
 $e_i(z', x) = (1 - s)(\lambda[G_i(x) - G_i(R(z'))]e_i(z, x_h) + (1 - \lambda)[e_i(z, x) - e_i(z, R(z'))])$
(17)

• If $x = x_h$

$$e_i(z', x_h) = (1 - s)(\lambda [1 - G_i(R(z'))]e_i(z, x_h) + (1 - \lambda)[e_i(z, x_h) - e_i(z, R(z'))]) + p_i(\theta_1(z), \theta_2(z))u_i(z, x)$$
(18)

Hence, total EU_i flows and the separation rates SR_i are

$$EU'_{i} = s \times e_{i}(z, x_{h}) + (1 - s)(\lambda G_{i}(R(z'))e_{i}(z, x_{h}) + (1 - \lambda)e_{i}(z, R(z')))$$
(19)

$$SR'_i = EU'_i/e_i(z, x_h) \tag{20}$$

and the implied law of motion for unemployment is

$$u'_i(z', x) = u_i(z, x) + EU'_i + (1 - p_i(\theta_1(z), \theta_2(z)) * u_i(z, x)$$
(21)

4 Quantitative Implications for the U.S. Economy

This section presents the quantitative properties of the model. First, it lays out the method to compute the stochastic dynamic equilibrium of the model. Second, it discusses the calibration of parameter values in the model, which is consistent with the empirical evidence for the U.S. labor market. Third, there is an evaluation of the model by comparing the business cycle moments from the model with the actual data. Finally, the section presents the results of a counterfactual experiment. The experiment studies whether the labor outcomes of the discriminated group would have been different had they had a relatively better hiring probability.

4.1 Solution Method

The model consists of the free entry condition (equation 11), the vacancy equation (equation 12), the two surplus equations (equation 13 for $i \in \{1, 2\}$), the (un)employment dynamics (equation 21), and the driving process (equation 3).

To find a numerical solution to the model, a preliminary step is the representation of stochastic elements in grids. The aggregate productivity z will evolve according to a Markov chain $\{z, \Psi\}$ with state-space $\{z_1, ..., z_I\}$ and $(I \times I)$ transition matrix Ψ with elements $\psi_{rj} = \mathbb{P}\{z' = z_j | z = z_r\}$. To this end, I apply the Rouwenhorst (1995) method for finite state Markov-chain approximations of AR(1) processes. In comparison with the more standard method of Tauchen (1986), the Rouwenhorst method has been found to generate accurate approximations to highly persistent stochastic processes ($\rho > 0.9$), which is typical in macroeconomic time series (Kopecky and Suen, 2010).

As in Fujita and Ramey (2012), G(x) is taken to be truncated lognormal and it is approximated by a discrete distribution with support $\{x_1, ..., x_M\}$, satisfying $x_1 = 1/M$, $\Delta = x_m - x_{m-1} = x_M/M$ and $x_M = x_h$. Thus, the associated probabilities $\{\gamma_1, ..., \gamma_M\}$ are

$$\gamma_{i} = \begin{cases} G(x_{i} + \Delta/2) & if \quad i = 1\\ G(x_{i} + \Delta/2) - G(x_{i} - \Delta/2) & if \quad i = 2, ..., M - 1\\ 1 - G(x_{i} - \Delta) & if \quad i = M \end{cases}$$

Value Function Iteration Numerical solutions are obtained via backward substitution. To illustrate, let z_r be the prevailing state, and let $\theta_i^T(z)$ and $S_i^T(z, x)$ be the functions obtained after *T* iterations for the two type of agents $i \in \{1, 2\}$. Thus, at iteration T + 1, the functions θ_i and S_i are updated according to the following equations

Match surplus

$$S_{i}^{T+1}(z_{r}, x_{m}) = \max\left\{0, z_{r}x_{m} - b + \beta(1-s) \left(\lambda \sum_{j=1}^{I} \sum_{n=1}^{M} \psi_{rj} \gamma_{n} S_{i}^{T}(z_{j}, x_{n})\right) + \beta(1-s) \left((1-\lambda) \sum_{j=1}^{I} S_{i}^{T}(z_{r}, x_{m})\right) - \beta(1-\eta) p_{i}(\theta_{1}^{T}(z_{r}), \theta_{2}^{T}(z_{r})) \sum_{j=1}^{I} S_{i}^{T}(z_{j}, x_{h})\right\}$$

where p_i is defined in equation (2).

• Market tightnesses $\theta_1^{T+1}(z_r)$ and $\theta_2^{T+1}(z_r)$ are $I \times 1$ vectors from the solution to the system of nonlinear equations formed by the free-entry condition 11 and the vacancy condition 12

$$0 = -c + \beta \eta \sum_{i=1}^{2} \sum_{j=1}^{I} q_i \left(\theta_1^{T+1}(z_r), \theta_2^{T+1}(z_r) \right) \psi_{rj} S_i^T(z_j, x_h)$$

$$0 = \left(\theta_1^{T+1}(z_r) / \theta_2^{T+1}(z_r) \right) - \left(u_2(z_r) / u_1(z_r) \right)$$

where $q_i = p_i / \theta_i$ and u_i is defined in equation (21).

4.2 Calibration

The model is calibrated to match aggregate labor market statistics of the U.S. economy. Only a subset of the model parameters are directly assigned, and the rest are estimated by simulated method of moments. I do not target moments describing the cyclicality or persistence of labor markets by groups, preserving these as outcomes by which the model can be evaluated. Table 3 summarizes the parameter values.

Parameter	Description	Value	Notes				
Panel A. Assigned Parameters							
β	Discount rate	0.9992	Interest rate 4% ; Shimer (2005);				
			Kydland and Prescott (1982)				
b	Unemployment payoff	0.71	Hall and Milgrom (2008)				
$ ho_z$	persistence of productivity	0.99	Labour productivity (BLS)				
σ_{z}	sd productivity innovations	0.003	Labour productivity (BLS)				
N_1	Population share of group 1	90%	White/Black share in the labor market				
π	Degree of hiring	1.38	Kuhn and Chancí (2019)				
	discrimination		(from resume studies)				
λ	Probability of changing	0.085	Fujita and Ramey (2012)				
	idiosyncratic productivity						
σ_x	S.D. log idiosyncratic	0.18	Separation rate 2.2% (CPS)				
	productivity						
Panel B. Estimates- Simulated Method of Moments							
ν	Employer's bargaining power	0.5408	See text				
S	Exogenous separation rate	0.0353	see text				
С	Vacancy creation cost	0.5296	See text				

Table 3: Calibration: Parameter Values.

Notes: This table presents the parameter values used in the model. Panel A presents the externally calibrated parameters. Panel B presents the internally calibrated parameters. Moments used to calibrate v, s and c are the average job-finding rate of 50 percent (Shimer, 2005), the inflow rates to unemployment of 2.2 percent (Fujita and Ramey, 2012), and the mean aggregated unemployment rate of 6.2 percent in CPS.

Externally calibrated parameters The calibration is weekly, and many features are standard in the literature. The value of the time discount factor is set to $\beta = 0.9992$ to accord with an annual risk-free interest rate of 4 percent (Kydland and Prescott, 1982; Shimer, 2005). The value of unemployment *b* consists of several elements: unemployment insurance benefits, home production, the value of leisure, and expenditures saves by not working. This parameter is somewhat controversial in the literature because it critically affects the simulation results. To illustrate, low values of *b* (e.g, 0.4 in Shimer, 2005) generate large surpluses and also low volatility of labor market variables. Conversely, a high value (e.g., 0.955 in Hagedorn and Manovskii, 2008) generates more volatility. As in Hall and Milgrom (2008), I set *b* equal to 0.71, which is in the middle of the range of values.

In the CPS the ratio of blacks to whites among labor force participants is roughly 1 to 9; thus, the share of the population of type 1 is set to $N_1 = 0.9$. The persistence and standard deviation of the aggregated productivity process are set to match the empirical behavior of labor productivity⁶, thus, $\rho_y = 0.99$ and $\sigma_{\epsilon} = 0.003$. As in Fujita and Ramey (2012), the idiosyncratic shocks are independent draws from a lognormal distribution occurring on average every quarter. Thus, the arrival rate of the match-specific productivity shock is $\lambda = 0.085$, and the standard deviation of the distribution is set to $\sigma_x = 0.18$.

Finally, as in Kuhn and Chancí (2019), the degree of hiring discrimination π is set to 1.38. This value represents a median estimate of the value in resume studies focusing on African-Americans in the US (Baert, 2018). Also, it is close to the point estimate of 1.49 in the seminal research of Bertrand and Mullainathan (2004).

Internally calibrated parameters The remaining three parameters are estimated by simulated method of moments. I use three targeted moments: (i) the mean of the unemployment rate; (ii) the mean of the job-finding rate; and (iii) the mean of the separation rate. The value for the average unemployment is 6.2 percent. This is the same value used in Kuhn and Chancí (2019), and represents a long-run mean of the unemployment rate. The average job-finding rate is targeted to 50 percent, which is in the range of values used in the literature (i.e., 45% in Shimer et al., 2005). The mean of the separation rate is targeted to match the mean of the monthly inflow rate to unemployment in the CPS of 2.2 percent. Assuming that two-thirds of all separations are

⁶U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Real Output Per Person [PRS85006163], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PRS85006163, January 5, 2020.

endogenous, this number represents a mean weekly rate of about 0.34 percent (Fujita and Ramey, 2012; Cairo and Cajner, 2018). There are as many parameters as there are targeted moments. Table 4 presents the list of targeted moments and model generated counterparts. The associated estimates are in Panel B of Table 3.

The estimated value for the employer's bargaining power ν is 0.5408 (first row of panel B in Table 3), which is well within the range of typical values cited in the literature. Moreover, this estimate is closer to the value of 0.5 that represents a symmetric Nash-bargaining sharing rule (see, for instance, Petrosky-Nadeau and Wasmer, 2017, Ch. 2). The estimated value for the exogenous separation rate *s* is 0.0353, which is comparable to the figures in the Fujita-Ramey data. Finally, the estimate for the vacancy creation cost is 0.5296, which is above some values cited in the literature (for instance, Fujita and Ramey, 2012, use a low value of 0.17).

Table 4:	Targeted	moments.
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Moment	Target	Model output
Mean unemployment rate	0.062	0.0615
Mean job-finding rate	0.50	0.59
Mean separation rate	0.034	0.038

Notes: This table presents targeted values used to estimate the parameters in panel B of Table 3. Column 3 shows the final values obtained from simulated method of moments.

4.3 **Business Cycle Moments**

This subsection presents an evaluation of the model by comparing the business cycle moments from the model with those from the CPS data. I first report several moments for the aggregate economy, and then present the results of the model by groups.

Results at the Aggregate Level Table 4 shows that, by design, the simulation results match the empirical means of the unemployment rate, the job-finding rate, and

the separation rate. Moreover, to perform a detailed evaluation of the model, I employ aggregate statistics for the US labor market reported in Fujita and Ramey (2012). The figures are based on CPS data and are already adjusted by time aggregation and other issues associated with the survey. Table 5 presents both the empirical statistics using CPS (panel A) and the results obtained from the model after 1,000 simulations (panel B). The rows present the volatility, the correlation with productivity, and the autocorrelations for seven labor outcomes: unemployment u_t , job-finding rate JFR_t , the flow of workers from employment to unemployment EU_t , vacancies v_t , and market tightness v_t/u_t .

Overall, the results show that the model performs reasonably well at the aggregate level. For instance, the figures in panel B have the same sign as in panel A. When comparing statistical moments by labor outcomes, as expected, some simulation results are closer to their empirical counterparts than others. On the one hand, the model underpredicts the volatility of unemployment, and the empirical standard deviation of unemployment is about nine times greater than the value generated by the model. This last result, however, is a common characteristic in most search-andmatching models, and it is well-known in the literature. On the other hand, the model performs well in predicting the volatility of the simulated separation rates. The model reproduces about 50 percent of the empirical value. It is also worth highlighting that the observed separation rate is not, in fact, constant, which supports the inclusion of endogenous separation rates in this paper. Finally, the model also succeeds in capturing the correlations between productivity and the labor market outcomes, being again an relevant case the separation rate - which is something missing in models with constant separation rate.

Labor Outcome Dynamics across Worker Groups Table 6 shows the model's results for each group. Mean values and standard deviations are in percentage points. Although the main interest of this paper is on the cyclical movements, or standard deviations, there are some results in terms of the mean values that are worth mentioning first. The model can generate differences in unemployment and job-finding rates across groups. In particular, with an unemployment rate of 6.45, group 2 has a 0.32 percentage point higher unemployment rate than group 1. Likewise, it takes workers of group 2 longer to find a job if they are unemployed, with a gap in job-finding rates of about 5 percent.

In terms of the standard deviations, the results in Table 6 show that the cyclical

	u_t	JFR_t	UE_t	SR_t	EU_t	v_t	v_t/u_t
Panel A. US Data							
σ_X	0.096	0.077	0.042	0.058	0.052	0.126	0.218
$cor(p_t, X_t)$	-0.460	0.369	-0.337	-0.535	-0.521	0.564	0.527
$cor(X_t, X_{t-1})$	0.926	0.804	0.416	0.631	0.560	0.920	0.930
Panel B. Model							
σ_X	0.011	0.097	0.024	0.029	0.024	0.010	0.209
$cor(p_t, X_t)$	-0.978	0.994	-0.560	-0.738	-0.609	0.994	0.998
$cor(X_t, X_{t-1})$	0.745	0.754	0.455	0.534	0.438	0.754	0.754

Table 5: US Data and Model: Moment Properties.

Notes: σ_X means the standard deviation of the variable *X*; $cor(p_t, X_t)$ means correlation between labor productivity *p* and *X*; $cor(X_t, X_{t-1})$ is the correlation between X_t and X_{t-1} . Panel A presents empirical values from Table 1 in Fujita and Ramey (2012). Panel B shows the results for aggregated variables obtained from the model in Section 3. Simulated data are quarterly averages of weekly series, logged and HP filtered, with smoothing parameter 1,600. Each replication computes simulated statistics from a sample of 200 quarterly observations. Reported statistics are averages over 1,000 replications.

X_t	u_t		JF	R_t	SR_t		
Group	(1)	(2)	(1)	(2)	(1)	(2)	
μ _X (%)	6.13	6.45	59.56	54.61	3.88	3.80	
σ_X (%)	1.06	1.16	0.95	1.11	0.30	0.28	
$cor(p_t, X_t)$	-0.98	-0.42	0.99	0.40	-0.66	-0.54	
$cor(X_t, X_{t-1})$	0.69	0.71	0.74	0.74	0.28	0.58	

Table 6: Second Moment Properties by Groups.

Notes: This table presents the simulation results from model in Section 3 by groups. μ_X is the mean of the variable X; σ_X means the standard deviation of the variable X; $cor(p_t, X_t)$ means correlation between labor productivity p and X; $cor(X_t, X_{t-1})$ is the correlation between X_t and X_{t-1} . Simulated data are quarterly averages of weekly series, logged and HP filtered, with smoothing parameter 1,600. Each replication computes simulated statistics from a sample of 200 quarterly observations. Reported statistics are averages over 1,000 replications.

movements in the unemployment and job finding rates are larger for group 2. Group 2's unemployment and job-finding rate have standard deviations of 1.16 and 1.11, respectively, whereas the standard deviations of group 1's unemployment and job-finding are 1.06 and 0.95, respectively. This result means that fluctuations in group 2's unemployment rate, for instance, exceed group 1's by about 10 percent. Conversely, the model predicts that there are larger differences -if any- in separation rates for group 1.

Finally, as mentioned in the calibration, I do not target moments describing the cyclicality or persistence of the labor outcomes by groups. I compare the model's results with those in the empirical section 2. Thus, consider a numerical example in which aggregate unemployment increases. Based on the point estimate from the regression model using CPS data (Table 2), during a severe recession in which average unemployment rises by 5 percent, the unemployment rate for black people increases close to 2 percentage points stronger than for white people. The calibrated model implies that, in such a recession, the difference in unemployment rates increases by 1.6 percentage points, thus accounting for about 80 percent of the empirically measured gap. Furthermore, the model does not predict significant differences in job-finding rates or for separation rates.

4.4 Counterfactual Experiment

The model in this paper is designated to be suitable for counterfactual policy analysis. A relevant experiment is a reduction in hiring discrimination, whether the labor outcomes of black people would have been different had they had a relatively higher hiring chance.

Table 7 presents the model's results if cutting the amount of hiring discrimination in half, $\pi = 1.19$. Overall, when comparing the figures in this table with the results in Table 6, there is an improvement in the group 2's labor outcomes. The mean unemployment rate decreases from 6.45 to 6.31 percent and the gap in the unemployment volatility basically disappears. Likewise, the gap in the job-finding rates increases by 2 percent, from 54.6 to 56.6 percent. Conversely, results for separation rates remain almost the same.

X_t	<i>u</i> _t		JF	R_t	SR_t	
Group	(1)	(2)	(1)	(2)	(1)	(2)
μ_X (%)	6.13	6.31	59.34	56.60	3.88	3.81
σ_X (%)	1.13	1.13	0.95	1.04	0.31	0.26

Table 7: Counterfactual Experiment: Setting $\pi = 1.19$.

Notes: This table presents the results of a counterfactual experiment. Parameter π in the model is reduced in half.

5 Conclusions

Resume studies have found that black workers have lower callback rates for job interviews than white workers. Starting from this evidence on racial discrimination during the hiring process, in this paper, I present a search-and-matching model to study the business cycle consequences, in terms of unemployment, job-finding and separation rates, of hiring discrimination against black Americans. The research is an extension of the work in Kuhn and Chancí (2019) by introducing endogenous separations. In summary, the resulting model has three components: (i) heterogeneous agents that face different hiring probabilities; (ii) labor market frictions; and (iii) endogenous separations. Thus, the model provides a quantitative mapping from the degree of hiring discrimination into differences in labor market outcomes. I define the equilibrium in terms of vacancies and unemployment, explain the numerical solution method, and calibrate the model to match the aggregate labor market statistics of the U.S. economy.

Adding endogenous separations makes the model more realistic and also improves the capability to match the volatility of unemployment. Although the model has a significant deviation to the model in Kuhn and Chancí (2019), and also uses an alternative numerical method, the central conclusions of these authors remain valid. That is, in addition to the effects on the level of unemployment, hiring discrimination has a sizeable adverse impact on the business cycle behavior of unemployment rates.

Using numerical results of hiring probabilities from resume studies as an input to the model, I find that the model replicates about 80 percent of the excess of unemployment volatility for black workers found in the CPS data. Furthermore, results from a counterfactual exercise - whether the labor outcomes of black people would have been different had they had a relatively higher hiring chance, suggest a significant improvement in the unemployment volatility of black workers.

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